Perfect Competition in an Oligopoly (including Bilateral Monopoly)

In honor of Martin Shubik*

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Abstract

We show that if limit orders are required to vary smoothly, then strategic (Nash) equilibria of the double auction mechanism yield competitive (Walras) allocations. It is not necessary to have competitors on any side of any market: smooth trading is a substitute for price wars. In particular, Nash equilibria are Walrasian even in a bilateral monopoly.

Keywords: Limit orders, double auction, Nash equilibria, Walras equilibria, perfect competition, bilateral monopoly, mechanism design

JEL Classification: C72, D41, D42, D44, D61

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*It is a pleasure for us to dedicate this paper to Martin Shubik who founded and developed (in collaboration with others, particularly Lloyd Shapley) the field of Strategic Market Games in a general equilibrium framework. This research was partly carried out while the authors were visiting IIASA, Laxenburg and The Institute for Advanced Studies, Hebrew University of Jerusalem. Financial support of these institutions is gratefully acknowledged.

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1 Introduction

As is well-known Walrasian analysis is built upon the Hypothesis of Perfect Competition, which can be taken as in Mas-Colell (1980) to state: "...that prices are publicly quoted and are viewed by the economic agents as exogenously given". Attempts to go beyond Walrasian analysis have in particular involved giving "a theoretical explanation of the Hypothesis itself" (Mas-Colell (1980)). Among these the most remarkable are without doubt the 19th century contributions of Bertrand, Cournot and Edgeworth (for an overview, see Stigler (1965)). The Cournot approach was explored intensively, in a general equilibrium framework, in the symposium issue entitled "Non-cooperative Approaches to the Theory of Perfect Competition" (Journal of Economic Theory, Vol. 22 (1980)).
The features common to most of the symposium articles are:

(a) The strategies employed by the agents are of the Cournot type, i.e., consist in quoting quantities.

(b) The (insignificant) size of any agent relative to the market is the key explanatory variable for the tendency of strategic behavior to approximate perfect competition and, in its wake, to lead to Walrasian outcomes (Mas-Colell (1980), p.122).

The extension of pure quantity strategies from Cournot's partial equilibrium model of oligopoly to a general equilibrium framework, however, does raise questions. Underlying the Cournot model is a demand curve for the particular market under consideration which enables the suppliers to relate quantities, via prices, to expected receipts. If such a close relationship is not provided by the market, then it seems more natural to us that an agent will no longer confine himself to quoting quantities, i.e., to pure buy-or-sell market orders. To protect himself against "market uncertainty - or illiquidity, or manipulation by other agents [1]", he will also quote prices limiting the execution of those orders, consenting to sell q units of commodity j only if its price is p or more, or buy $\tilde{q}$ units only if its price is $\tilde{p}$ or less. By sending

[1] to quote Mertens (2003)

multiple orders of this kind an agent can approximate any monotone demand or supply curve in a market by a step function, as was done in Dubey (1982, 1994). Here we go further and give each agent full manoeuvrability. He places a continuum of infinitesimal limit-price orders, which in effect enables him to send any monotone, continuous demand or supply curve for each commodity[2]. The upshot is a striking result: provided only that all commodity markets are "active" (i.e. there is positive trade in them), and no matter how thin they are, *strategic (Nash) equilibria* (SE) coincide – in outcome space – with *competitive (Walras) equilibria* (CE). Our result thus provides a rationale, based on strategic competition, for Walrasian outcomes even in the case of a bilateral monopoly. This brings it in sharp contrast to Dubey (1982, 1994), where it was necessary to allow for price wars via competition on both sides of each market (in the sense of there being at least two active buyers and two active sellers for each commodity) in order to conclude that SE are CE [3]. The key point of our paper is that *smooth trading is a substitute for price wars and yields perfect competition.* A monopolist may be in sole command of his own resource, but nevertheless he will be reduced to behaving as if he had cut-throat rivals, once smooth trading sets in[4].

For better perspective, we consider two different versions of our model. In the first, our focus is on an oligopoly in which each trader tries to exert monopoly power. Thus a seller may be (or else feel to be) in sole control of commodity j. Given an (inverse) price-quantity demand curve $D_j(q)$ for j, he will try to appropriate the entire consumer surplus under the demand

[2]It must be emphasized that our model is based on *decentralized* markets, and is therefore an order-of-magnitude simpler than that of Mertens (2003), where cross-market limit orders are permitted. SE form a large superset of CE in Mertens' model – for instance, the SE of Shapley's windows model (see Sahi and Yao (1989)) are also SE there.

[3]Indeed, in Dubey's model, the coincidence of SE and CE fails drastically if there is a monopolistic agent in any market. In particular, in a bilateral monopoly, every individually rational and strategically feasible allocation is sustained by SE!

[4]A related phenomenon was analyzed in Coase (1972) (and following Coase (1972), a long line of literature, see e.g., Bulow (1982), Gaskins (1974), Schmalensee (1979)). There, too, a monopolist was shown to forfeit his power, but this happened in the setting of durable goods which could be sold sequentially over time to infinitely patient customers. In our model the monopolist loses power even with perishable goods which are traded at one instant of time.

curve D_j by perfect price discrimination; i.e., selling first to the highest-priced buyer of j , then to the next highest, and so on. Similarly, a buyer of commodity j, who feels himself to be in a monopolistic position facing several sellers, will first take the cheapest offer of j, then the next cheapest, etc., in order to appropriate the entire producer surplus in market j. Indeed, in the extreme case of a bilateral monopoly, perfect price discrimination of this kind is to be expected. Our equilibrium point EP captures what happens when monopolistic expectations spill over into a general oligopolistic framework. The notion of EP is akin to Walrasian equilibrium, with the important difference that prices are not fixed from the outside by an imaginary auctioneer, but vary with the size of the trade and are set by the agents themselves, each of whom realizes and exerts his ability to influence prices. Therefore we think that EP is an interesting concept in its own right.

In the second version we turn to a standard market game, as in Dubey (1982) and Dubey (1994). The market functions like a stock exchange, with all higher ask (or, lower bid) prices serviced before a new purchase (or, sale) order is taken up. In contrast to Version 1, each agent here is grimly realistic and realizes that the prices he will get are apropos his own quotations, not the best going; and thus he calculates his expenditure (or, revenue) as the integral under his own demand (or, supply) curve.

To accommodate economies in which CE consumptions could occur on the boundary, it becomes needful here to introduce a "market maker" who has infinitesimal inventories of every good, and stands ready to provide them if sellers renege on their promises of delivery [5]. It turns out that, at our SE, the market maker is never active. But it is important for agents to imagine his presence when they think about what they could get were they to unilaterally deviate.

Though the two versions are built on quite different behaviorial hypotheses, we find their equilibria (the EP and the SE) lead to the same outcomes, namely Walrasian.

Our model shares some of the weaknesses of the Walrasian model. In particular, since it is based on the static concept of a strategic equilibrium,

[5] If we restrict attention to interior CE, the market maker can be dispensed with. See Section 3.9 .

our model does not address the question of what dynamic forces bring the equilibrium about and ensure that individual strategic plans become jointly compatible. But it goes beyond the Walrasian notion in at least three important ways:

(a) It is not assumed that the economic agents face perfectly elastic supply and demand curves.

(b) Prices are not quoted from the outside, but set by the agents themselves.

(c) Strategies of the individuals (i.e. supply and demand curves submitted to the market) need not be based on their true characteristics (preferences and endowments) – indeed, in equilibrium, they never are!

2 The First Version: Walrasian Outcomes via Equilibrium Points

Let $N = \{1, \ldots, n\}$ be the set of agents who trade in k commodities. Each agent $i \in N$ has an initial endowment $e^i \in \mathbb{R}^k_+ \setminus \{0\}$ and a preference relation $\succsim_i$ on $\mathbb{R}^k_+$ that is convex, continuous and monotonic (in the sense that $x \geq y$, $x \neq y$ implies $x \succ_i y$). We assume that $\sum\limits_{i \in N} e^i \gg 0$, i.e. every named commodity is present in the aggregate.

An agent may enter a market either as a buyer or a seller, but not both (although he may switch roles), and submit to each of the k commodity markets a marginal (inverse) demand or supply curve. Formally, let

$$M^+ = \{f : \mathbb{R}_+ \to \mathbb{R}_{++} |\ f \text{ is continuous and non-decreasing}\}$$
$$M^- = \{f : \mathbb{R}_+ \to \mathbb{R}_{++} |\ f \text{ is continuous and non-increasing}\}.$$

Then a *strategic choice* σ^i of agent i is given by

$$\sigma^i = (\sigma^i_1, \ldots, \sigma^i_k \,|\, \sigma^i_j = d^i_j \in M^- \ \textit{or}\ \sigma^i_j = s^i_j \in M^+,\ \text{for}\, j = 1, \ldots, k)$$

In the interpretation $d^i_j(q^i_j)$ is the price at which agent i is willing to buy an infinitesimal, incremental unit of commodity j, once his level of purchases has reached q^i_j. The supply curve has an analogous meaning. Denote $\sigma \equiv (\sigma^1, \ldots, \sigma^n)$ and let S^σ_j, D^σ_j be the aggregate[6] supply, demand curves.

We suppose that agent i acts under the optimistic conjecture that he can exert perfect price discrimination, i.e., that he can sell (buy) starting at the highest (lowest) prices quoted by the buyers (sellers). This means that agent i calculates his receipts (or expenditures) on the market j as the integral, starting from 0, under the curve D^σ_j (or S^σ_j). The generally non-convex budget set $B^i(\sigma)$ is then obtained by the requirement that (perceived) expenditures do not exceed (perceived) receipts, i.e.,

$$B^i(\sigma) = \{e^i + t \mid t \in \mathbb{R}^k, e^i + t \in \mathbb{R}^k_+, \sum_{j=1}^{k} E^\sigma_j(t_j) \leq \sum_{j=1}^{k} R^\sigma_j(t_j)\}$$

where

$$E^\sigma_j(t_j) = \int_0^{t_j} S^\sigma_j(q)dq \quad \text{if } t_j > 0,\ 0 \text{ otherwise},$$

$$R^\sigma_j(t_j) = \int_0^{|t_j|} D^\sigma_j(q)dq \quad \text{if } t_j < 0,\ 0 \text{ otherwise}.$$

(Note that $t^i_j > 0$ ($t^i_j < 0$) means that i buys (sells) j . In the sequel we will drop the integration variable dq .)

The collection of strategic choices σ will be called an *equilibrium point* (EP) if there exist trade vectors $t^1, \ldots, t^n$ in $\mathbb{R}^k$ such that

(i) $\quad e^i + t^i$ is $\succsim_i$ -optimal on $B^i(\sigma)$ for $i = 1, \ldots, n$

(ii) $$\sum_{i=1}^{n} t^i_j = 0 \text{ for } j = 1, \ldots, k$$

[6]The aggregation is *horizontal*. In other words, taking *price* to be the independent variable, each s^i_j is a monotonically increasing *correspondence*; and so S_j can be viewed to be the sum (over i) of these correspondences, which is also montonically increasing. Reverting to quantity as the independent variable, S_j is a monotonically (weakly) increasing function. D_j is similarly defined.

(iii) $$\sum_{i:t_j^i>o} t_j^i = sup\{q_j \mid S_j^\sigma(q_j) \leq D_j^\sigma(q_j)\} \text{ for } j = 1, \ldots, k$$

Conditions (i) and (ii) require that agents optimize and that markets clear. Condition (iii) says that no trade can be enforced, i.e., it stops when the (marginal) supply price for the first time exceeds the demand price; and, at the same time, in equilibrium all trades compatible with the submitted strategies are actually carried out.

An EP will be called *active* if there is positive trade in each market.

First let us establish that at an active EP all trade $T_j := \sum_{i\,:\,t_j^i>0} t_j^i$ in any commodity j takes place at *one* price, p_j.

Lemma 1. *The curves S_j^σ and D_j^σ coincide and are constant on $[0, T_j]$ at any EP .*

Proof. For any j, let $G_j := \{i : t_j^i > 0\}$, $H_j := \{i : t_j^i < 0\}$. Then

$$
\begin{aligned}
\text{(1)} \qquad \sum_{i \in H_j} R_j^\sigma(t_j^i) &= \sum_{i \in H_j} \int_0^{|t_j^i|} D_j^\sigma \\
&\geq \int_0^{T_j} D_j^\sigma \\
&\geq D_j^\sigma(T_j) \cdot T_j \\
&\geq S_j^\sigma(T_j) \cdot T_j \\
&\geq \int_0^{T_j} S_j^\sigma \\
&\geq \sum_{i \in G_j} \int_0^{t_j^i} S_j^\sigma \\
&= \sum_{i \in G_j} E_j^\sigma(t_j^i).
\end{aligned}
$$

The third inequality follows from (iii); the other four follow from monotonicity of the supply and demand functions.

Hence

$$\sum_{i=1}^{n} R_j^\sigma(t_j^i) \geq \sum_{i=1}^{n} E_j^\sigma(t_j^i) \quad \text{for } j = 1, \dots, k. \tag{2}$$

From the monotonicity of preferences, and the fact that each agent has optimized, we have

$$\sum_{j=1}^{k} R_j^\sigma(t_j^i) = \sum_{j=1}^{k} E_j^\sigma(t_j^i) \quad \text{for } i = 1, \dots, n. \tag{3}$$

(2) and (3) together imply:

$$\sum_{i=1}^{n} R_j^\sigma(t_j^i) = \sum_{i=1}^{n} E_j^\sigma(t_j^i) \quad \text{for } j = 1, \dots, k. \tag{4}$$

From (4) it follows that all the inequalities in (1) must, in fact, be equalities. Therefore

$$S_j^\sigma(T_j) = D_j^\sigma(T_j) =: p_j \tag{5}$$

and

$$\int_0^{T_j} D_j^\sigma = p_j T_j = \int_0^{T_j} S_j^\sigma. \tag{6}$$

Since by (iii), $D_j^\sigma \geq S_j^\sigma$ on $[0, T_j]$ we get, from (6), and the monotonicity of D and S

$$D_j^\sigma = S_j^\sigma \text{ on } [0, T_j]. \tag{7}$$

□

In view of Lemma 1 we can talk not only of the allocation but also the prices produced at an active EP . These are the constant values of S_j^σ, D_j^σ on $[0, T_j]$ for $j = 1, \dots, k$. Note that these prices are positive by assumption.

Proposition 1. *The prices and allocation at an active equilibrium point are Walrasian.*

Proof. Let σ be an EP with trades $t^1, \ldots, t^n$ and prices p . We need to show that, for each i , $e^i + t^i$ is $\gtrsim_i$ -optimal on the set

$$B^i(p) := \{e^i + t : t \in \mathbb{R}^k, e^i + t \in \mathbb{R}^k_+,\ p.t = 0\}.$$

W.l.o.g. fix $i = 1$, put

$$\begin{aligned}
J_1 &:= \{j : t^1_j > 0\} \\
J_2 &:= \{j : t^1_j < 0\} \\
J_3 &:= \{j : t^1_j = 0\} \\
T_j &:= \sum_{i : t^i_j > 0} t^i_j \\
\delta_j &:= min[|t^1_j|, T_l : j \in J_1 \cup J_2, l \in J_3] \\
N_j &:= \{\alpha \in \mathbb{R} : |t^1_j - \alpha| < \delta_j\} \\
F_j &:= E_j - R_j
\end{aligned}$$

(Since the EP is active, $\delta_j > 0$). Now we claim, for $j = 1, \ldots, k$:

(8) F_j is continuously differentiable and strictly increasing on N_j and its derivative at t^1_j is p_j.

This follows from the continuity and strict positivity of S_j and D_j, and from Lemma 1 which implies:

(9) $F_j(q)$ coincides with $E_j(q) = p_j q$ if $j \in J_1,\ 0 \leq q \leq t^1_j$

(10) $F_j(q)$ coincides with $-R_j(q) = p_j q$ if $j \in J_2,\ t^1_j \leq q \leq 0$

(11) $F_j(q) = p_j q$ if $j \in J_3,\ q \in N_j$.

W.l.o.g. fix commodity $j = 1$. Since $F_1, \ldots, F_k$ are all strictly increasing and $\sum_{j=1}^{k} F_j(t_j^1) = 0$, and $F_1'(t_1^1) = p_1 > 0$, it follows from the implicit function theorem that there is a neighborhood V of $(t_2^1, \ldots, t_k^1)$ in $N_2 \times \ldots \times N_k$ such that if $(t_2, \ldots, t_k) \in V$ then there is a unique t_1 which satisfies the equation $F_1(t_1) + \ldots + F_k(t_k) = 0$. Thus we have an implicit function $G(t_2, \ldots, t_k) = F_1^{-1}(-F_2(t_2) - \ldots - F_k(t_k))$ defined on V which is clearly continuously differentiable. Finally the point $t^1 = (t_1^1, \ldots, t_k^1)$ belongs by construction to the smooth hypersurface $M = \{(G(t_2, \ldots, t_k), t_2, \ldots, t_k) : (t_2, \ldots, t_k) \in V\} \subset B^1(\sigma)$ and, by (8), the tangent plane H to M at this point has normal p .

Since we are at an EP, $e^1 + t^1$ is $\succsim_1$-optimal on $(e^1 + M) \cap \mathbb{R}_+^k$. Suppose that there is some $x \in H_+ := (e^1 + t^1 + H) \cap \mathbb{R}_+^k$ such that $x \succ_1 e^1 + t^1$. By continuity of $\succsim_1$ we can find a neighborhood Z of x (in $\mathbb{R}_+^k$) with the property: $y \in Z \Rightarrow y \succ_1 e^1 + t^1$. But since M is a smooth surface there exists a point y^* in Z, such that the line segment between y^* and $e^1 + t^1$ pierces $e^1 + M$ at some point $z^* \in (e^1 + M) \cap \mathbb{R}_+^k$ (see Fig.1). By convexity of $\succsim_1$, we have $z^* \succ_1 e^1 + t^1$, contradicting that $e^1 + t^1$ is $\succsim_1$-optimal on $(e^1 + M) \cap \mathbb{R}_+^k$. We conclude that $e^1 + t^1$ is $\succsim_1$-optimal on H_+ . But we have $e^1 \in H_+$ (simply set trades to be zero, i.e., pick $-t^1$ in H). Therefore, in fact, $H_+ = B^1(p)$. Since the choice of $i = 1$ was arbitrary, the proposition follows. □

.... Insert Figure 1 here!................

Proposition 2. *If the trades $t^1, \ldots, t^n$ and prices $p \gg 0$ are Walrasian, then they can be achieved at an EP.*

Proof. For any i let

$$J_1^i = \{j : t_j^i > 0\}$$
$$J_2^i = \{j : t_j^i < 0\}$$
$$J_3^i = \{j : t_j^i = 0\}$$
$$f_j^i = \text{any strictly decreasing function with } f_j^i(t_j^i) = p_j$$
$$g_j^i = \text{any strictly increasing function with } g_j^i(t_j^i) = p_j$$

and consider

$$s_j^i(x) = \begin{cases} 0 & \text{if } j \in J_1^i \cup J_3^i \\ max\{p_j, g_j^i(x)\} & \text{if } j \in J_2^i \end{cases}$$

$$d_j^i(x) = \begin{cases} 0 & \text{if } j \in J_2^i \cup J_3^i \\ min\{p_j, f_j^i(x)\} & \text{if } j \in J_1^i \end{cases}$$

Then it is readily checked that these strategies constitute an EP and produce the trades $t^1, \ldots, t^n$ at prices p. □

3 Strategic Market Games: Implementing Walras Equilibria with an Infinitesimal Market Maker

The foregoing analysis can be recast in terms of strategic Nash equilibria SE of a market game. We shall adopt the perspective of the mechanism design literature. The aim is to *prescribe* a market mechanism which Nash-implements the Walras correspondence. Of course Maskin's well known results (see Maskin (1999)) imply that this is impossible unless the domain of economies is restricted so as to ensure that the final Walrasian consumption is strictly in the interior of $\mathbb{R}_+^k$ for each agent. But we shall place no such restrictions here. (We take them up in Section 3.9 .) Instead we shall imagine a "market maker" who has inventory of $\varepsilon_j > 0$ units of each commodity $j \in K \equiv \{1, ..., k\}$ and who is ready to bring them to market if any seller reneges on his promise to deliver, thereby giving the buyers *something* to look forward to. *No matter how small* $\varepsilon = (\varepsilon_1, ..., \varepsilon_k)$ is, so long as it is positive, CE are implemented as SE. Our analysis thus sheds some light on Maskin's result. It shows that the *breakdown of the desired mechanism is not drastic, but of the size of an arbitrarily small* ε. Moreover, given the leeway of ε, we do not have to imagine esoteric mechanisms (such as those which entail the strategic announcement of unboundedly large integers – see, again, Maskin

(1999)) : the simple double auction, in its smooth incarnation, will do the job.

It should be noted that our infinitesimal market maker is not called upon to take any action at the SE of our strategic game. He only lurks in the background. It is enough for every agent i to *believe* that the market maker would make available the infinitesimal inventory ε, were i to unilaterally deviate from SE and thereby trigger a situation in which some sellers of commodity j are unable to deliver on their promises. The belief in the market maker ensures that he is never called upon to prove his existence[7]. We feel that this role of the market maker is not pure mathematical gimmickry, but has counterparts in the real world. One need only think of a broker who has a small inventory of company stocks from the past, and who is willing to make them available to his buyer clients to mitigate seller default on deliveries.

The point of our analysis in Section 3 is not only that Maskin's result on the impossibility of Nash-implementation of non-interior CE can be overcome with an infinitesimal market maker. Nor is it to add to the list of abstract mechanisms which implement the Walras correspondence. Many such have already been presented (see, e.g., Hurwicz (1979), Hurwicz, Maskin, and Postlewaite (1980), Postlewaite (1985), Schmeidler (1980), Giraud and Stahn (2003) – all of which, incidentally, require at least three agents, in addition to interior CE, and bypass the case of a bilateral monopoly). We are instead inspired by the fact that the double auction has a long and rich history, not only in academia, but in real market processes (see Friedman and Rust (1993) for an excellent survey). Our analysis reveals that a "smoothened" version of the double auction will make for efficiency and help to break monopoly power. It thereby implies that, if the "price-jumps" permitted in bidders' strategies are reduced by mandate of the auction-designer, every such reduction will come with efficiency gains. To that extent, we hope that our analysis below will also be of some interest to applied economists who are concerned with the general properties of double auctions.

[7]Indeed, we can reinterpret the scenario in terms of "refined" SE of the game without the market maker, relegating the market maker to ε-trembles in the refinement. (See Section 3.7).

3.1 The Subeconomy $\mathcal{E}_J$

It will be useful to define subeconomies $\mathcal{E}_J$ of the whole economy $\mathcal{E} = (e^i, \succeq_i)_{i \in N}$ for any subset $J \subset K \equiv \{1, ..., k\}$ of commodities. For a vector $y \in \mathbb{R}^K$, denote $y_J \equiv (y_j)_{j \in J} \in \mathbb{R}^J$. Then the set of agents in $\mathcal{E}_J$ is $\{i \in N : e^i_J \neq 0\}$, with endowments e^i_J and preferences $\succeq_{i,J}$ on $\mathbb{R}^J_+$ given by the rule: $z \succeq_{i,J} y$ iff $(z, e^i_{K \setminus J} \succeq_i (y, e^i_{K \setminus J})$.

3.2 Strategy Sets

There is a market for each commodity, as before. An agent must enter each market either as a buyer or as a seller (and, for simplicity, not both). If i enters as a buyer for commodity j, he must submit a strategic demand function $d^i_j : \mathbb{R}_+ \longrightarrow \mathbb{R}_{++}$ which is weakly decreasing, *and* smooth (i.e., continuously differentiable)[8]. The interpretation is that i is willing to pay $\int_0^x d^i_j(t)dt$ units of "fiat money" in order to purchase x units of commodity j. (There is no endowment of money in our model. But imagine that each agent can borrow money without limit at zero interest rate, from a bank in the background, prior to commodity trade and that the loan is due after trade.)

In the same vein, if i enters market j as a seller he must submit a strategic supply function $s^i_j : \mathbb{R}_+ \longrightarrow \mathbb{R}_{++}$ which is weakly increasing, smooth and (only for ease of presentation) satisfies $\lim_{x \to \infty} s^i_j(x) = \infty$. In *addition*, i must put up $\tilde{\theta}^i_j > 0$ (with $\tilde{\theta}^i_j \leq e^i_j$) as "collateral" for his intention to sell j. Finally we stipulate that each agent must enter at least one market as a seller. Thus the strategy set $\sum^i$ of agent i is given by

$$\begin{aligned} \textstyle\sum^i \quad = \quad & \{(d^i_j, s^i_j, \tilde{\theta}^i_j)_{j \in K} : \text{one, and only one,} \\ & \text{of } d^i_j, s^i_j \text{ is } \phi \text{ for every } j;\ s^i_j \neq \phi \text{ for at} \\ & \text{least one } j;\ 0 < \tilde{\theta}^i_j \leq e^i_j \text{ if } s^i_j \neq \phi; \\ & \tilde{\theta}^i_j = 0 \text{ if } s^i_j = \phi\} \end{aligned}$$

where the functions d^i_j, s^i_j satisfy the conditions mentioned.

[8] If i does not enter market j as a buyer (or, seller), we write $d^i_j = \phi$ (or, $s^i_j = \phi$). The symbol ϕ means that the curve referred to is missing.

3.3 Outcomes

The market does a sequence of computations based on the N-tuple $(\sigma_i)_{i\in N} \in \mathop{\mathbf{X}}_{i\in N} \sum^i$ of submitted strategies, in order to impute commodity trades and monetary payments to the agents. The idea is simple. Trade is allowed up to the intersection of the aggregate demand and supply curves, with priority accorded to the higher (lower) priced buyer (seller). But two kinds of default can occur. An agent may be unable or unwilling to deliver the goods he is called upon to. Or else he may go into a budget deficit when his purchases exceed the proceeds of his sales. We require that either kind of default be severely punished. One may think here of the obvious stipulation (as we ourselves do in Section 3.9) that the entire initial endowment of any defaulter is confiscated (as in, e.g., Peck, Shell, and Spear (1992), or Weyers (1999) or Giraud and Stahn (2003)); or that (as in Dubey, Geanakoplos, and Shubik (2005)) sufficient disutility is inflicted on him by extraneous (unmodeled) means . This does make for a very swift description of the mechanism. But it presupposes that the market has knowledge of agents' private characteristics (endowments and utilities). Our aim here is to design a market process that is anonymous, in keeping with the mechanism design literature. This is why we have introduced collaterals. The rules of confiscation, spelled out below, operate only on what agents publicly send to market of their own strategic volition (and in full cognizance of the rules). The market does not know – or need to know – agents' private characteristics. It merely confiscates the entire collateral of any agent who defaults (either on delivery or on budget balance) and prevents him from trading. The confiscated goods are, of course, made available to buyers. The only subtlety is that default on delivery must be dealt with first, since this affects what buyers purchase, and thereby budget balances. For clarity we spell the process out:

Step 1 Compute the aggregate demand D_j and aggregate supply S_j for each $j \in K$ as before. (Those curves which are missing are naturally ignored in the aggregation. If $s^i_j = \phi$ for *all* i , then the aggregate supply S_j is also deemed missing and we write $S_j = \phi$. Similarly $D_j = \phi$ if $d^i_j = \phi$ for *all* i .)

Step 2 Compute the set $J \subset K$ of markets in which D_j and S_j intersect[9] (at, necessarily, a unique price p_j - see Figure 2).

Step 3 In each market $j \in J$, compute sales by agents until the price p_j, rationing proportionately quantities offered for sale at the margin price p_j in the event that there is excess supply at p_j (see Figure 2). Denote these sales $(\theta_j^i)_{i \in N}$. (Some θ_j^i could be zero, provided $s_j^i = \phi$ or $s_j^i(0) > p_j$.)

If $\theta_j^i > \tilde{\theta}_j^i$ for *some* $j \in J$ (i.e., i's collateral fails to cover his imputed sale θ_j^i at some market), then i is declared a "*defaulter*" and forbidden to trade across *all* markets, and his collateral is confiscated at *every* market that he submitted them to.

Step 4 At each $j \in J$, define

$$Q_j = \begin{cases} \sum\limits_{i \in N} \theta_j^i, \text{ if there is no seller-default at } j \\ \varepsilon_j + \sum\limits_{i \in N} \min\left\{\theta_j^i, \tilde{\theta}_j^i\right\} \text{ otherwise} \end{cases}$$

(Recall that ε_j is the market maker's infinitesimal inventory of commodity j). The market maker now allocates Q_j to buyers on D_j, starting at the highest price $D_j(0)$ in D_j and rationing proportionately the demand at the margin price $D_j(Q_j)$ if necessary (i.e., if there is excess demand at this price). Denote these purchases $(\varphi_j^i)_{i \in N}$. If i is already a defaulter in Step 3, he is ignored; otherwise his net debt is computed:

$$\Delta^i = \sum_{j \in J} \int_0^{\varphi_j^i} d_j^i(t)dt - \sum_{j \in J} \int_0^{\theta_j^i} s_j^i(t)dt$$

(For $d_j = \phi$ or $s_j^i = \phi$, the integral is taken to be zero.) If $\Delta^i > 0$, then again i is declared a "defaulter" and dealt with as before, i.e., his collateral is confiscated at every market in which he put them up and he is forbidden from trading.

.... Insert Figure 2 here!................

[9] At markets $j \in K \backslash J$, the intersection fails to occur either because S_j lies above D_j, or because one of the curves S_j or D_j is missing.

3.4 Payoffs

Agents $i \in N$ who are not defaulters (as in Step 3 or in Step 4) buy φ_j^i and sell θ_j^i in markets $j \in J$. They obtain payoff $u^i(x^i)$ where

$$x_j^i = \begin{cases} e_j^i + \varphi_j^i - \theta_j^i \text{ if } j \in J \\ e_j^i \text{ if } j \in j \in K \backslash J \end{cases}$$

Defaulting agents i obtain payoff $u^i(y^i)$ where

$$y_j^i = e_j^i - \tilde{\theta}_j^i \text{ for } j \in K.$$

This well defines a game Γ in strategic form on the player set N. By SE we shall mean a strategic (Nash) equilibrium in pure strategies of the game Γ.

3.5 Active SE are Walrasian

Define a market to be *active* in an SE if there is positive trade at that market.

Proposition 3. *At any SE with active markets J, all trade in $j \in J$, takes place at one price p_j. Moreover these prices and the final allocation constitute a CE of the economy $\mathcal{E}_J$.*

Proof. We will prove the proposition for the case $J = K$. (The same argument holds for any $J \subset K$ and the corresponding economy $\mathcal{E}_J$.)

First observe that by lowering d_j^i to $\tilde{d}_j^i$ so that $\tilde{d}_j^i(0) < S_j(0)$ and by raising s_j^i to $\tilde{s}_j^i$ so that $\tilde{s}_j^i(0) > D_j(0)$, any agent i can ensure that he does not trade and so end up consuming his initial endowment e^i. But if i defaults, his utility is less than that of e^i, since he loses his collateral in at least one market and purchases nowhere. We conclude that there is no default in an SE.

Next we assert that (at an SE) in each market j all trade must be taking place at the intersection price p_j. The proof of this is similar to that of Lemma 1. Indeed, no more than the money paid out by agent-buyers goes to agent-sellers, implying $\sum\limits_{i \in N} \Delta^i \geq 0$. But no default also implies $\Delta^i \leq 0$ for all $i \in N$. We conclude that $\Delta^i = 0$ for all $i \in N$. Now if any purchase took place above p_j or any sale below p_j in *some* market j, then (since purchases [or, sales] occur at prices $\geq$ [$or, \leq$] the intersection price at *every* market), we would have: total money paid out by agents across all markets > total money received by agents across all markets. This would imply $\Delta^i < 0$ for some i, a contradiction, proving our assertion.

Consider the bundles that an agent i can obtain via unilateral deviation from his own strategy at the SE. First suppose i is a buyer of commodity j at the SE.

Case 1 There exists at least one other active buyer of j at the SE, or else there is excess supply of j at the SE price $p_{j.}$

In this case, i can buy slightly more of j at the price p_j by simply demanding a slightly higher quantity at p_j. (The maneuver works for i even if he is the sole buyer of j and the sellers of j have no collateral left to back further sales. This is on account of the market maker who stands ready to make up for the sellers from his inventory, enabling i to buy a little more).

Case 2 Case 1 fails, i.e., i is the sole buyer of j and there is no excess supply of j at the SE price p_j.

In this case, i can demand a little more at a slightly higher price (i.e., raise the flat part of his demand curve, keeping it flat till it intersects S_j). Since S_j is continuously differentiable, the extra quantity purchased by i will vary smoothly with the rise in the intersection price. (The fact that i can indeed buy a little more is once again assured by the infinitesimal inventory of the market maker.)

By a similar argument, i can sell a little more of any commodity j' that he was selling at the SE, either at the same price or at a price that is slightly lower and varies smoothly with the extra quantity sold.

Clearly i can *reduce* his sale and purchase and get the same price as at the SE.

Thus it is feasible for i to enhance trade a *little* beyond his SE trade in a smooth manner. More precisely, he can get consumption bundles on a smooth ε-extension $M(\varepsilon)$ of the flat part of his achievable set of bundles (where the extension is computed using prices smoothly increasing/decreasing away from p_j in accordance with the D_j/S_j curves). The situation is depicted in Figure 1, with the curved bold line extended only slightly beyond the flat part, and representing $M(\varepsilon)$.

But the argument in the proof of Proposition 1 applies, *no matter how small the smooth extension* $M(\varepsilon)$ *may be*: if x is not optimal on i's Walrasian budget set, then there exists a point z^* on $M(\varepsilon)$ which yields more utility to i than x, contradicting that i has optimized. $\square$

Define an SE to be *active* if all markets are active in it. Then Proposition 3 implies

Proposition 4. *The prices and allocations at an active SE are Walrasian .*

Remark: It is evident that, *given our SE*, an agent cannot profit by a unilateral deviation to arbitrary piecewise-continuous monotonic demand and supply curves (e.g., kinked curves, or, worse, step-functions as in Dubey(1982)). This is so because, via such deviations, all he can accomplish is to buy more (or sell more) at prices at least as high (or at least as low) as the SE prices. But since the SE is a CE, he does not even have incentive to trade more at the SE prices (see Section 4.2 for details).

This shows a certain robustness of our SE. (But, of course, the SE must first emerge from a game with smooth strategies. That is the mechanism we have designed or *prescribed*. Our aim is *not* to *describe* any empirical process per se).

3.6 Walrasian outcomes are achieved at active SE

It is evident that Proposition 4 in fact holds if we allow agents to enter each market both as buyers and as sellers. The mechanism is well-defined, treating buy and sell orders as separate and disregarding the fact that they came from the same individual. Once we enhance the strategy sets in this manner, it is easy to establish along the lines of Proposition 2 :

Proposition 5. *The prices and allocations at any CE can be achieved at an active SE.*

Proof. Consider strategies in which every agent offers to sell his entire endowment at the CE prices (and to sell more at higher prices, as in the proof of proposition 2); and offers to buy his CE consumption bundle at the CE prices (and to buy more at lower prices). It is clear that these strategies constitute an active SE. □

While Proposition 5 is technically correct it can leave one feeling a little uneasy, because (as its proof makes evident) it is based on "wash sales", i.e., sale of a commodity by an agent who buys it back at the same price. However, the slightest transaction costs would eliminate such sales. Thus we develop Proposition 7 in the next section as an alternative to Proposition 5.

3.7 Refined Nash Equilibria

We now couch our results in terms of equilibrium refinement.

Fix the economy $(e^i, \succeq_i)_{i\in N}$ and let Γ_ε denote the strategic market game when the market maker has inventories $\varepsilon = (\varepsilon_1, ...\varepsilon_k) \in \mathbb{R}^K_{++}$ of the various commodities. Thus Γ_0 is the game without the market maker.

We shall say that an SE σ of Γ_0 is *refined* if there exist SE $\sigma(\varepsilon)$ of Γ_ε such that $\sigma(\varepsilon) \longrightarrow \sigma$ as $\varepsilon \longrightarrow 0$.

It is immediate that the market maker can be removed from the foreground and put into the ε-trembles of the refinement process, so that Proposition 4 may be reworded :

Proposition 6. *Active, refined SE of Γ_0 coincide in prices and allocations with the CE of the underlying economy $(e^i, \succeq_i)_{i \in N}$.*

It may be useful to compare[10] our refinement with that of other models, which also invoke an ε-market maker (see, e.g., Dubey and Shubik (1978) and its variants). In those models, agents must think of ε-perturbed games G_ε in which the market maker is trading up to ε in all markets, not just off the SE play but *in* SE. Typically G_ε has multiple SE, all of which are different from the candidate SE of the original game G_0 . The agents must coordinate beliefs on the same SE of G_ε as an approximation of the candidate SE. In contrast, in our model here, the supply and demand curves, and the collaterals, are throughout given objectively by the candidate strategies. They constitute an SE not only of the original game Γ_0 without the market maker, but also of all the perturbed games Γ_ε. The ε-market maker need only be conjectured in Γ_ε, off the fixed SE play, by an agent when he unilaterally deviates. Furthermore the conjecture is rudimentary: each agent i thinks – *without* varying the candidate strategies of the others – that an extra supply ε_j will be available at market j if collaterals fail to cover the sales. Thus, both at a conceptual and interpretational level, and as a mathematical device, our refinement is simplicity itself compared to that of Dubey and Shubik (1978)

Of course, the purpose in Dubey and Shubik (1978) is to distinguish bona-fide inactivity at markets from ad hoc inactivity. We sidestepped this in Proposition 6 by assuming that the SE are active to begin with. We now strengthen the notion of refinement to allow for inactive markets. Once again our notion is simpler in that no wholly new SE of the ε-perturbed game Γ_ε need be coordinated upon by the agents.

Imagine that, in our game Γ_ε, the market maker further endeavors to bolster trade by offering to buy (and, sell) up to $\tilde{\varepsilon}_j > 0$ units of commodity j at some common price $\tilde{p}_j$ and to buy (and, sell) more at smoothly decreasing (and, increasing) prices. Treating the market maker as a strategic dummy, and postulating that he creates the commodity and the money that the mechanism calls upon him to deliver, the game is well-defined even after some subset $J \subset K$ of markets are $\tilde{\varepsilon}_j - \tilde{p}_j$ – perturbed as described. We shall

[10]We are grateful to both the anonymous referees for suggesting this comparison.

say that an SE $\sigma(\varepsilon)$ of Γ_ε is "*-refined" if there exist $\tilde{\varepsilon}_j - \tilde{p}_j$ – perturbations of the inactive[11] markets in $\sigma(\varepsilon)$ that do not disturb the SE $\sigma(\varepsilon)$. It is then trivial to verify (using the convexity of preferences) that *-refined SE of Γ_ε coincide with the CE of $(u^i, \succeq_i)_{i\in N}$. (i.e., *-refinement eliminates the need to postulate activity in all markets in Proposition 5). Now say that an SE σ of Γ_0 is *strongly refined* if there exist *-refined SE $\sigma(\varepsilon)$ of Γ_ε such that $\lim \sigma(\varepsilon) \longrightarrow \sigma$ as $\varepsilon \longrightarrow 0$. Then we obtain:

Proposition 7. *Strongly refined SE of* Γ_0 *coincide in prices and allocations with the CE of* $(e^i, \succeq_i)_{i\in N}$

3.8 Strong Nash Equilibria

It can be checked that our SE are strong (i.e. no coalition of agents can by coordinatedly changing its strategies – assuming others fixed – Pareto-improve itself). The proof of this is similar to that of the analogous proposition in Dubey (1982), hence omitted.

3.9 Eliminating Collaterals and the Infinitesimal Market Maker

Define the economy $\mathcal{E} = (e^i, \succeq_i)_{i\in N}$ to be *interior* if, for all $i \in N$,

$$\left\{x \in \mathbb{R}_+ : x \succeq_i e^i\right\} \subset \mathbb{R}_{++}\,.$$

Such economies form a standard domain in the mechanism design literature (see, e.g., Maskin (1999)).

[11] A market can be inactive because agents have taken it into their heads to send crazy orders to it (with sellers asking for exorbitant prices and buyers offering absurdly low prices). On the other hand it may be open for business, quoting a single price at which the market maker is ready both to buy and to sell, and nevertheless remain inactive because agents are choosing voluntarily not to go there. The purpose of *-refinement is to rule out the first kind of inactivity (in which markets are arbitrarily "shut"), but allow for the second kind (in which markets are "open", though no one is coming there).

Let us now consider our mechanism exactly as before, but *without* the infinitesimal market maker and *without* collaterals. In lieu of this, suppose that a seller is *obliged* to deliver what the market calls upon him to do, so long as this does not exceed his (initial) endowment (as in Dubey (1982)); and that his entire endowment is confiscated if he goes into budget deficit. (We can now also drop the requirement that each agent enter at least one market as a seller).

Proposition 8. *Propositions 4 and 5 hold in the modified mechanism, without the infinitesimal market maker and without collaterals, provided the economy $\mathcal{E}$ is interior.*

Proof. This follows at once from our earlier arguments. The main point to note is that, at an SE, no agent will have sold all his endowment of any commodity. (He prefers not to trade rather than wind up with zero consumption in any component.) Thus if a buyer deviates at the SE to buy an infinitesimal amount more of some commodity j (albeit at prices that rise smoothly with the extra purchase), all the sellers of j will deliver. Our entire analysis therefore remains intact. □

4 Equilibrium Points – Revisited [12]

4.1 Pessimistic Expectations

The expectation of each agent that he can exert perfect price discrimination may appear unduly optimistic to some readers, especially when there are several rivals on every market. But our results remain intact even under the opposite behavioral assumption of extreme pessimism. To be precise, assume agents' strategic demand and supply curves are smooth as in Section 3.2 . Define

$$Q_j^\sigma \equiv inf\{q : D_j^\sigma(q) \geq S_j^\sigma(q)\}$$

(with $Q_j^\sigma = 0$ if either $D_j^\sigma = \phi$, or $S_j^\sigma = \phi$, or S_j^σ is strictly above D_j^σ). Let each agent i think that he can buy q units of commodity j, where $0 \leq q \leq Q_j^\sigma$,

[12] We thank both referees for queries that led to this section.

for the expected expenditure $\widehat{E}_j^\sigma(q) = \int_0^q D_j^\sigma$. (To buy more than Q_j^σ, he would need to enhance his d_j^i , in order to increase Q_j^σ.) Similarly, he can sell $0 \le |q| \le Q_j^\sigma$ for the expected receipt $\widehat{R}_j^\sigma(q) = \int_0^{|q|} S_j^\sigma$. (Remember: $q \le 0$ if it represents a sale). Now define the pessimistic budget set $\widehat{B}^i(\sigma)$ just like $B^i(\sigma)$ with $E_j^\sigma(q)$ and $R_j^\sigma(q)$ replaced by $\widehat{E}_j^\sigma(q)$ and $\widehat{R}_j^\sigma(q)$. After this define EP exactly as before.

We submit that Propositions 1 and 2 still hold. To see this first note that

$$\sum_{i \in H_j} \int_0^{|t_j^i|} S_j^\sigma \le \int_0^{T_j} S_j^\sigma, \quad \text{and} \quad \sum_{i \in G_j} \int_0^{t_j^i} D_j^\sigma \ge \int_0^{T_j} D_j^\sigma\,;$$

and then verify Lemma 1 by rereading the proof with $\widehat{E}_j^\sigma$, $\widehat{R}_j^\sigma$ in place of R_j^σ, E_j^σ (reversing the chain of inequalities). Next (given an active EP) note that each agent can access a smooth extension $M(\varepsilon)$ of the flat part of his EP budget set, exactly as in Case 2 of the proof of Proposition 3 . Thus, as argued there, the EP is a CE.

Note that the expectations we have taken are indeed very pessimistic. It would be an intermediate, and probably more realistic, hypothesis that (as in Version 2) agent i expends $\int_0^{t_j^i} d_j^i$ (or, receives $\int_0^{|t_j^i|} s_j^i$), so long as his trade does not exceed Q_j^σ. In this case, too, our result remains valid. To see this, note that (see footnote 6)

$$\sum_{i \in H_j} \int_0^{|t_j^i|} s_j^i = \int_0^{|T_j|} S_j^\sigma\,, \quad \text{and} \quad \sum_{i \in G_j} \int_0^{t_j^i} d_j^i = \int_0^{T_j} D_j^\sigma,$$

and verify Lemma 1, and the rest of the argument, as above.

One may also imagine other intermediate expectations that are sandwiched between our polar extremes of optimism and pessimism , which leave our results intact, by preserving Lemma 1 (via either direction on the chain of inequalities) and the ability to access $M(\varepsilon)$ [13].

[13] This will happen if, for instance, buyers (sellers) expect that the variable price at

4.2 Stability of Equilibrium Points

Let σ be an active EP with prices p. Let each agent i conjecture that he can unilaterally vary his trades in some abstractly given set $\Xi^i(\sigma)$ which satisfies two constraints:

(a) he can buy (or, sell) more than his EP purchase t^i_j (or, sale $|t^i_j|$) at an average price that is at least (or, at most) the EP price p_j;

(b) he does not go into budget deficit for any trade in $\Xi^i(\sigma)$.

Then, as in the remark following Proposition 4, it is immediate that his EP trade t^i is $\succeq^i$-optimal in $\Xi^i(\sigma)$. Indeed t^i is $\succeq^i$-optimal in the set $B^i(p)$ of feasible *Walrasian* trades (since the EP is a CE). But $\Xi^i(\sigma) \subset B^i(p)$, since in $B^i(p)$ he can trade freely at *fixed* prices p , which is more favorable than what the constraints (a) and (b) permit. Hence t^i is also $\succeq^i$-optimal in $\Xi^i(\sigma)$. In particular it follows that an active EP of Version 1 is an SE of the market game of Version 2.

which they incrementally purchase (sell) quantities is above (below) the intersection price, i.e., agents are *consistently* pessimistic. (The degree of their pessimism may vary, with the most negative buyers placing themselves at the extreme left segment of the aggregate demand curve, others in the middle, and the least negative at the extreme right segment culminating in the intersection.). Similarly if there is consistent – albeit variable – optimism among agents with all purchases (sales) expected below (above) the intersection price, Lemma 1 remains valid (with the inequalities reversed), as does the accessibility of $M(\varepsilon)$.

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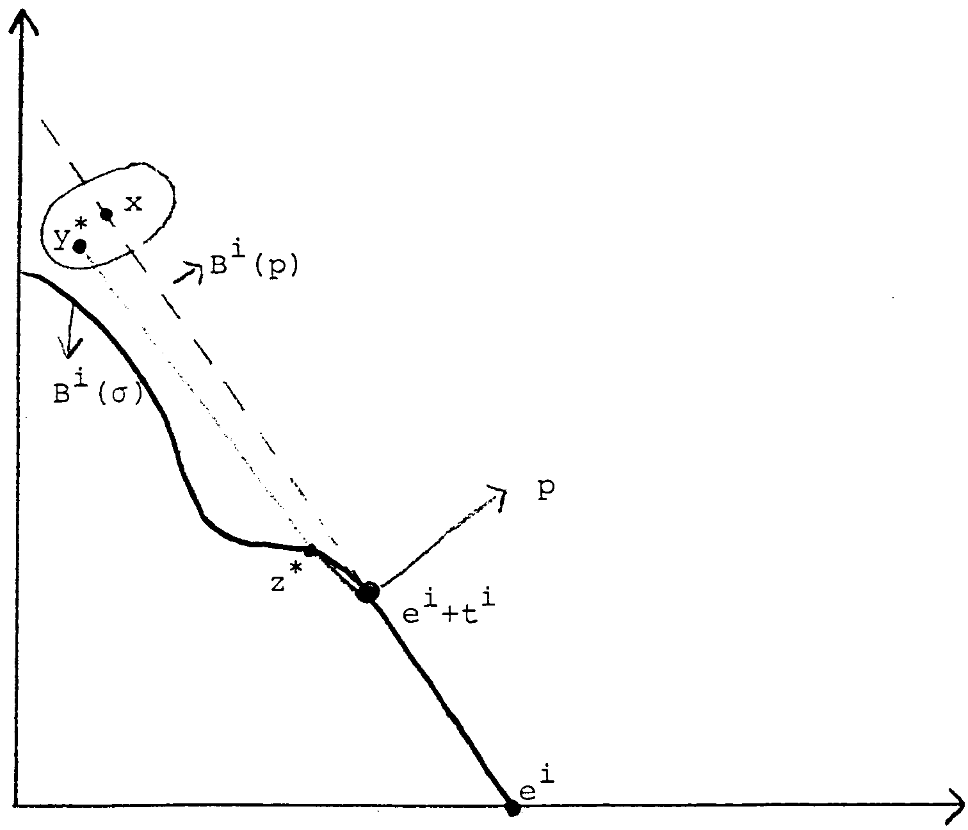
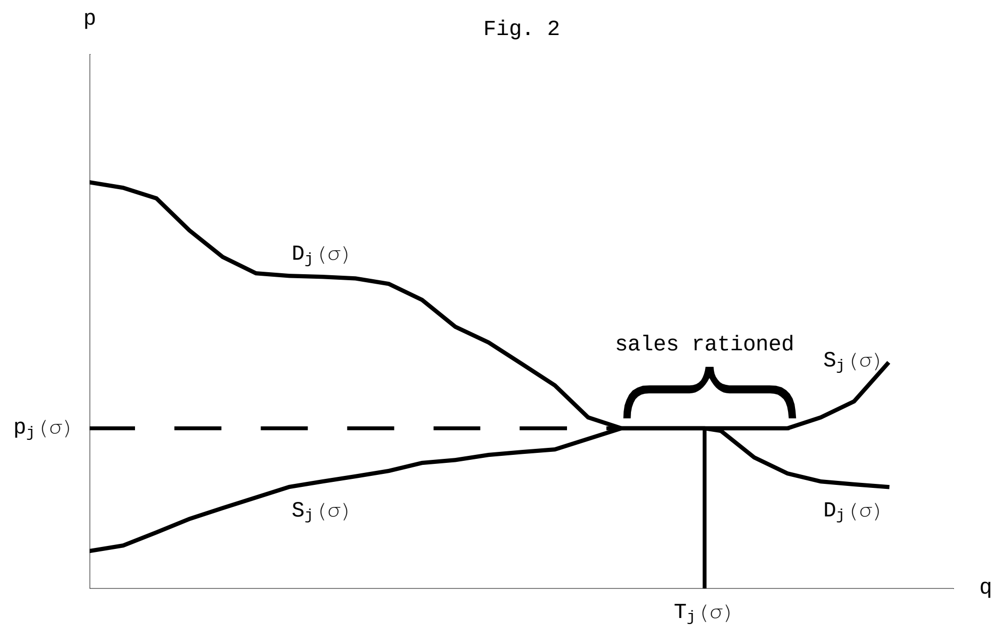


Fig. 1

Fig. 2


p
$D_j(\sigma)$
sales rationed
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q
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